

Coulomb glory in low-energy antiproton scattering by heavy nucleus: screening effect of vacuum polarization

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Abstract

Backward scattering of antiprotons by bare uranium is studied theoretically for antiproton energies within the interval 100 eV – 1 keV. A marked maximum of the differential cross section in the backward direction (Coulomb glory) at some energies of the incident particle is revealed. The effect is due to the screening properties of the vacuum polarization potential and can be regarded as a manifestation of the vacuum polarization in non-relativistic collisions of heavy particles. Experimental observation can become feasible with new facilities for antiproton and ion research at GSI.

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The project FAIR (Facility for Antiproton and Ion Research) at GSI in Darmstadt will give an opportunity to get high-intensity antiproton beams at energies between 30 MeV and 300 keV at a magnetic storage ring and at energies between 300 keV and 20 keV at an electrostatic storage ring. It will be possible to decelerate the antiprotons to ultra-low eV energies by means of heavy ion trap facilities. This will make accessible a large variety of new atomic collision experiments, such as investigations of the antiproton scattering. These investigations open new opportunities in observation of the Coulomb glory effect, which was predicted in Refs. [1, 2]. The phenomenon results in a prominent maximum of the differential cross section (DCS) in the backward direction at some energy of the incident particle, provided the interaction with the target is represented by a screened Coulomb attraction potential (the pure Rutherford cross section has a smooth minimum at 180° irrespectively of the energy).

In the previous paper [3] we investigated the backward scattering of antiprotons by highly charged and neutral uranium ($Z = 92$). It was shown that the Coulomb glory effect takes place due to screening of the nuclear Coulomb attraction by the electrons. In collisions of antiprotons with bare uranium, the Coulomb glory is also present because of the effect of vacuum polarization (VP), which was accounted for in Ref. [3] within the Uehling approximation. Observation of the Coulomb glory in collisions of antiprotons with bare uranium nuclei can be of particular interest since the screening property of the VP potential in non-relativistic collisions of heavy particles can be manifested. In the present paper we study the influence of the exact one-loop VP potential on the backward scattering of antiprotons with bare uranium. The calculations have been performed in the framework of semiclassical and quantum theory. Atomic units ($\hbar = e = m_e = 1$) are used in the paper.

The non-relativistic scattering theory can be applied if the kinetic energy of the antiproton is as low as a few hundreds of electron volts. We use the partial wave expansion of the scattering amplitude $A(\theta)$,

$$A(\theta) = \exp(2i\delta_0^c) \left[-\frac{\nu}{2k \sin^2 \theta/2} \exp \left(-2i\nu \ln \sin \frac{\theta}{2} \right) + \frac{1}{k} \sum_{l=0}^{\infty} (-1)^l (2l+1) P_l(\cos \theta) \times \exp(i\delta_l^s) \sin \delta_l^s \frac{(1-il/\nu) \dots (1-i/\nu)}{(1+il/\nu) \dots (1+i/\nu)} \right] \quad (1)$$

where k is the momentum of the antiproton, $\nu = -Zm_{\bar{p}}/k$ is the Coulomb parameter ($m_{\bar{p}}$

is the antiproton mass) and $P_l(\cos \theta)$ are the Legendre polynomials. Note that the total scattering amplitude is a sum of the pure Coulomb amplitude and the contribution due to short-range (non-Coulomb) terms in the scattering potential. The phase shifts δ_l^s are the differences between the total phase shifts δ_l and the Coulomb phase shifts δ_l^c

$$\delta_l^s = \delta_l - \delta_l^c. \quad (2)$$

DCS is related to the scattering amplitude as follows:

$$\frac{d\sigma}{d\Omega} = |A(\theta)|^2. \quad (3)$$

The phase shifts δ_l^s can be calculated by means of the variable phase method [4, 5, 6] without solving the radial Schrödinger equation. Within this approach, the variable phase $\delta_l^s(k, r)$ is a solution of a first-order differential equation. In our case, this differential equation can be written as

$$\begin{aligned} \frac{d}{dr} \delta_l^s(k, r) &= -2m_{\bar{p}} k v(r) r^2 \\ &\times [\cos \delta_l^s(k, r) F_l(k, r) - \sin \delta_l^s(k, r) G_l(k, r)]^2. \end{aligned} \quad (4)$$

Here $F_l(k, r)$ and $G_l(k, r)$ are the regular and irregular Coulomb wave functions, respectively [7], and $v(r)$ is the short-range part of the scattering potential $V(r)$:

$$v(r) = V(r) + \frac{Z}{r}. \quad (5)$$

The initial condition for solving (4) is $\delta_l^s(k, 0) = 0$. The phase shift is the limit of $\delta_l^s(k, r)$ as $r \rightarrow \infty$:

$$\delta_l^s = \lim_{r \rightarrow \infty} \delta_l^s(k, r). \quad (6)$$

For the energies of the order of a few atomic units the Coulomb parameter is large ($|\nu| \gg 1$), and the motion of the antiproton can be described in the framework of the quasiclassical approximation [8]. Then the phase shifts δ_l^s can be presented as a difference of the two integrals which correspond to the phases of the quasiclassical wave functions in the total scattering potential and in the Coulomb potential:

$$\begin{aligned} \delta_l^s &= \lim_{R \rightarrow \infty} \left\{ \int_{R_0}^R dr \sqrt{2m_{\bar{p}} (E - V(r)) - \frac{(l + 1/2)^2}{r^2}} \right. \\ &\quad \left. - \int_{R_C}^R dr \sqrt{2m_{\bar{p}} \left(E + \frac{Z}{r} \right) - \frac{(l + 1/2)^2}{r^2}} \right\}, \end{aligned} \quad (7)$$

where R_0 and R_C are the classical turning points for the two motions, respectively. In our calculations we applied both the variable phase method and the quasiclassical approximation and found that the results are very close to each other for the energy range under consideration (100 eV – 1 keV).

The total potential $V(r)$ experienced by the antiproton due to electromagnetic interaction is represented by a sum of the potential of a finite nucleus and the VP potential:

$$V(r) = V_n(r) + V_{VP}(r). \quad (8)$$

The potential of a finite nucleus is given by

$$V_n(r) = - \int d^3r' \frac{\rho_n(r')}{|\mathbf{r} - \mathbf{r}'|}. \quad (9)$$

Here ρ_n is the nuclear charge density, normalized to the nuclear charge number Z . We employ the Fermi-like nuclear charge distribution

$$\rho_n(r) = \frac{N_0}{1 + \exp[(r - r_0)/a]} \quad (10)$$

where the parameter a is equal to $2.3/(4 \ln 3)$ fm, the parameters r_0 and N_0 are derived from the root-mean-square nuclear charge radius and the normalization condition [9].

The VP potential is conveniently represented as a sum of the Uehling and the Wichmann–Kroll (WK) potential:

$$V_{VP}(r) = V_{Uehl}(r) + V_{WK}(r). \quad (11)$$

The Uehling potential is given by the lowest-order term in the expansion of the one-electron-loop vacuum polarization in powers of the Coulomb electron-nucleus interaction. According to the Furry theorem, this term contains one Coulomb interaction in the vacuum loop and is ultraviolet divergent. It becomes finite after charge renormalization. The renormalized expression for the Uehling potential is given by (see, e.g., [9, 10])

$$V_{Uehl}(r) = -\frac{2}{3rc^2} \int_0^\infty dr' r' \rho_n(r') \int_1^\infty dt \left(1 + \frac{1}{2t^2}\right) \frac{\sqrt{t^2 - 1}}{t^3} \times [\exp(-2c|r - r'|t) - \exp(-2c(r + r')t)] \quad (12)$$

where c is the speed of light in vacuum.

The Wichmann–Kroll potential $V_{WK}(r)$ accounts for the higher-order terms in the expansion of the vacuum loop in powers of the Coulomb electron-nucleus interaction [11].

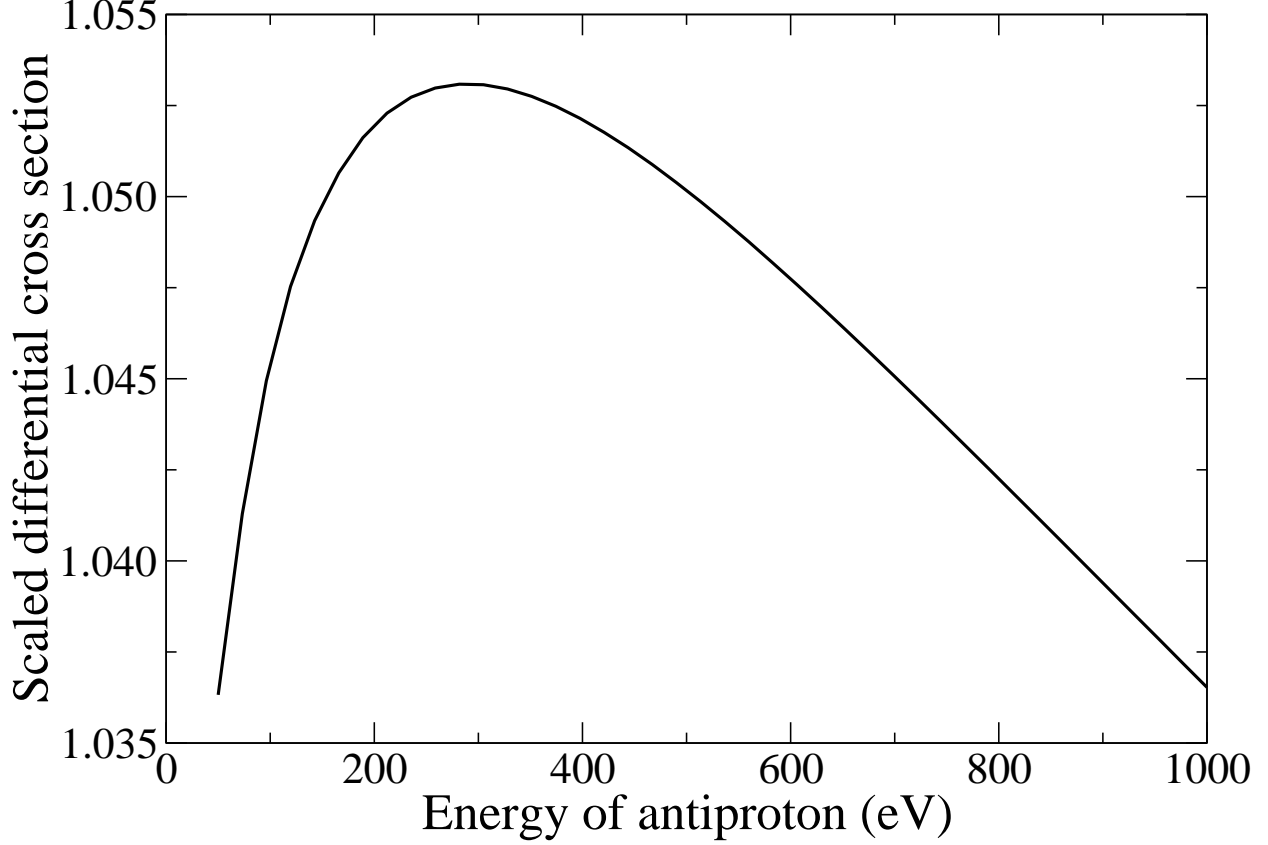


FIG. 1: Scaled DCS $d\tilde{\sigma}/d\Omega$ (14) at 180° for the total scattering potential (8) vs the antiproton energy.

Although the WK potential is finite, the regularization is still required in the lowest-order non-zero term due to a spurious gauge-dependent piece of the light-by-light scattering contribution. The spurious term disappears if the WK potential is calculated by summing up the partial-wave differences between the full one-loop contribution and the unrenormalized Uehling term [12, 13, 14, 15]. The calculation formula for the WK potential can be written as [10, 16]:

$$\begin{aligned}
V_{\text{WK}}(r) = & \frac{2}{\pi} \sum_{\kappa=\pm 1}^{\pm\infty} |\kappa| \int_0^\infty d\omega \int_0^\infty dr_1 r_1^2 \int_0^\infty dr_2 r_2^2 \frac{1}{\max(r, r_1)} \\
& \times V_n(r_2) \sum_{i,k=1}^2 \text{Re}\{F_\kappa^{ik}(i\omega, r_1, r_2)[G_\kappa^{ik}(i\omega, r_1, r_2) - F_\kappa^{ik}(i\omega, r_1, r_2)]\}.
\end{aligned} \tag{13}$$

Here κ is the relativistic angular momentum quantum number, $G_\kappa^{ik}(i\omega, y, z)$ and $F_\kappa^{ik}(i\omega, y, z)$ are the radial Dirac components of the partial-wave contributions to the bound and free electron Green's functions, respectively.

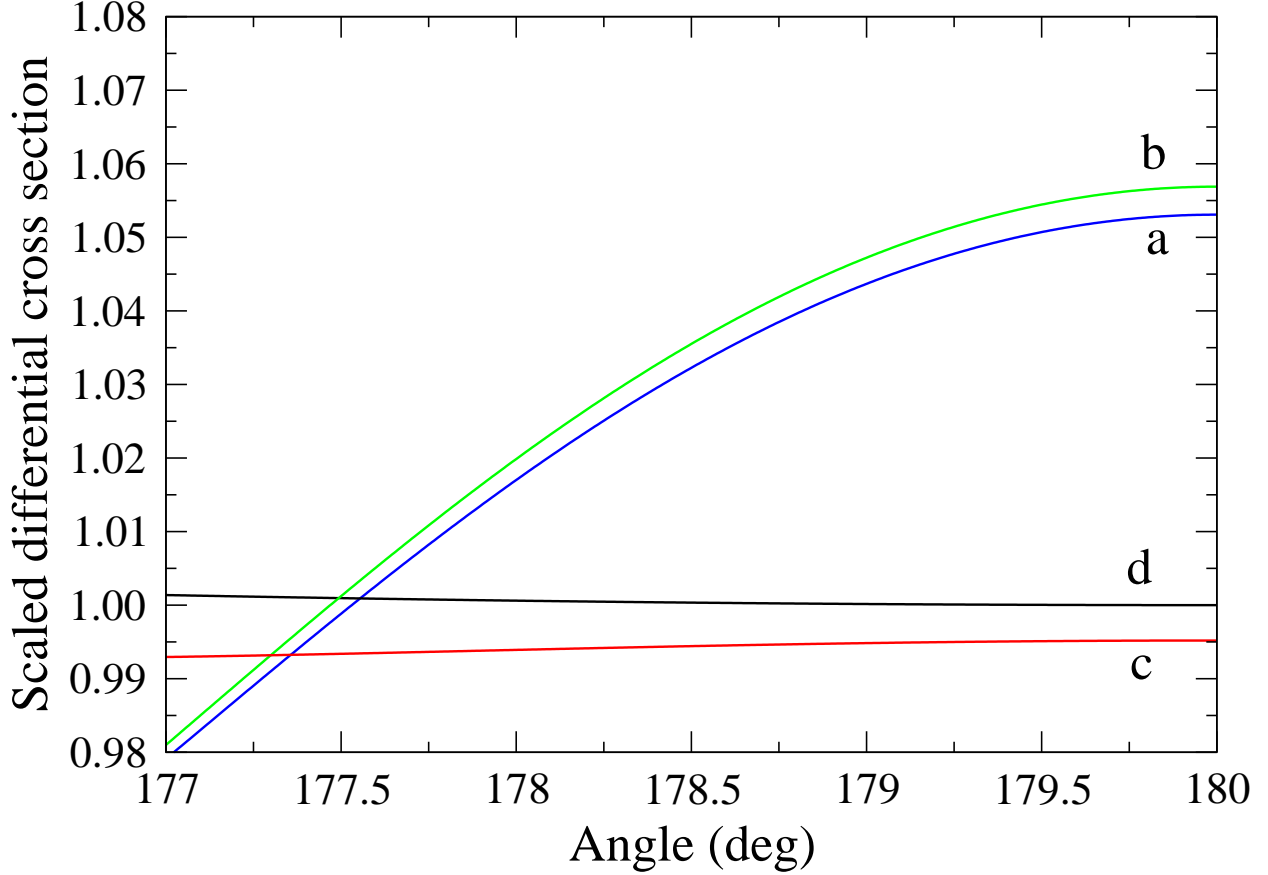


FIG. 2: Scaled DCS $d\tilde{\sigma}/d\Omega$ (14) for the energy of the antiproton 300 eV. (a) DCS for the total scattering potential (8); (b) DCS for the potential including the finite nucleus and Uehling contributions only; (c) DCS for the finite nucleus potential only; (d) DCS for the pure Coulomb potential (scaled Rutherford cross section).

We have calculated DCS of elastic scattering of antiprotons by bare uranium nuclei in the energy range from 100 eV to 1 keV. To make the results at different energies comparable, the differential cross section has been scaled according to

$$\frac{d\tilde{\sigma}}{d\Omega} = \left(\frac{4E}{Z}\right)^2 \frac{d\sigma}{d\Omega}. \quad (14)$$

The scaled Rutherford cross section does not depend on the antiproton energy and the nuclear charge number and is equal to unity at $\theta = 180^\circ$. The value of $d\tilde{\sigma}/d\Omega$ at $\theta = 180^\circ$ represents the ratio of the DCS for the scattering by the uranium nucleus and the corresponding Rutherford DCS and can serve as a quantitative measure of the Coulomb glory effect.

In figure 1 we show the scaled DCS as defined by Eq. (14) at $\theta = 180^\circ$ as a function

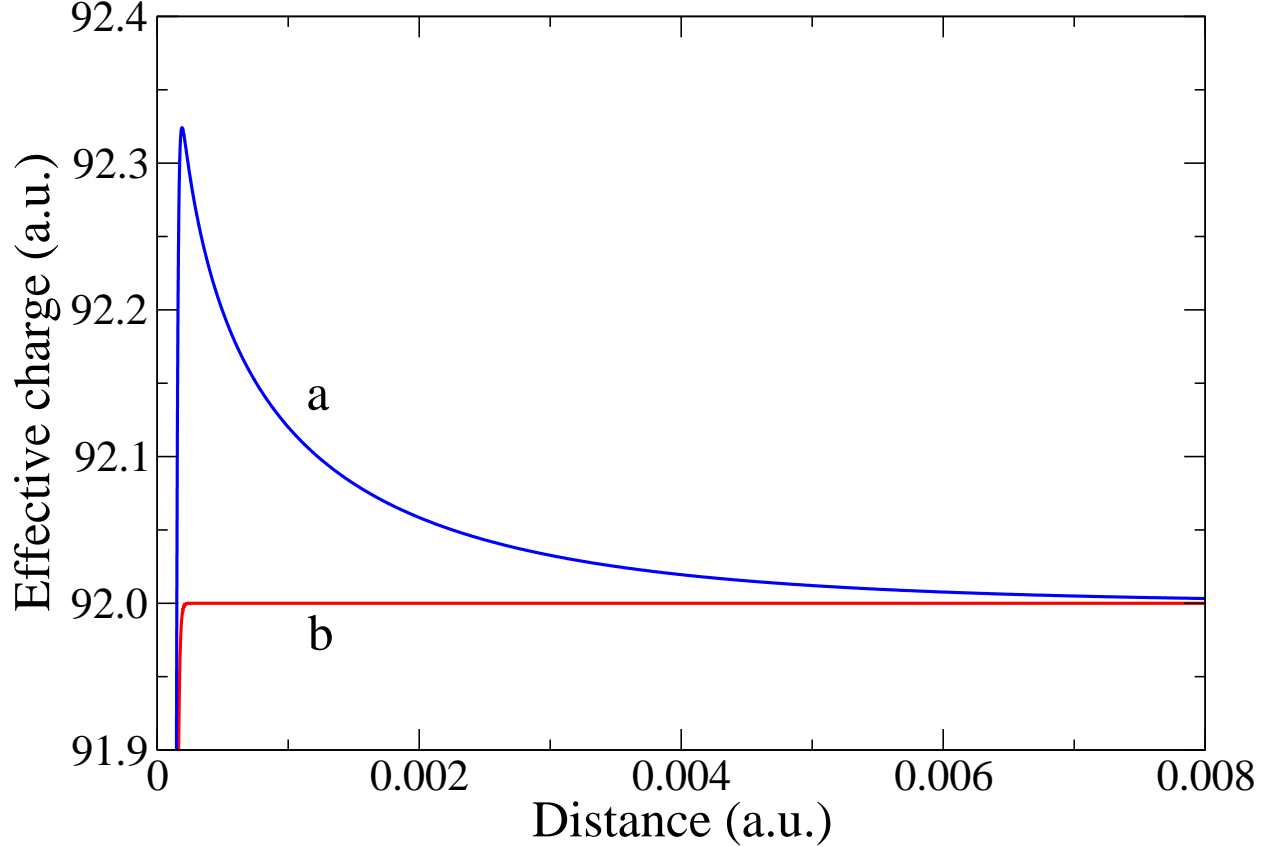


FIG. 3: The effective charge $Z_{\text{eff}}(r) \equiv -rV(r)$ as a function of r . (a) $Z_{\text{eff}}(r)$ for the total scattering potential (8); (b) $Z_{\text{eff}}(r)$ for the finite nucleus potential only.

of the antiproton energy. As one can see, the uranium nucleus DCS is larger than the corresponding Rutherford DCS for all the energies in the range, but the maximum Coulomb glory effect ($d\tilde{\sigma}/d\Omega(180^\circ) = 1.053$) is reached at the antiproton energy of about 300 eV. It should be noted that the maximum of DCS at $\theta = 180^\circ$ is a result of constructive interference of different angular momentum contributions to the total scattering amplitude. The more angular momenta in equation (1) interfere constructively, the larger is the amplitude and the cross section. At the same time, the individual contributions (that are the phase shifts δ_l^s) must not be too small. On the other hand, the number of partial waves with large enough phase shifts depends on the range of the scattering potential. That is why the three terms in the total scattering potential (8) are not equally important for the Coulomb glory effect. To estimate the influence of the different terms in the total scattering potential (8) on the Coulomb glory, we have also calculated DCS with partial scattering potentials. For the antiproton energy 300 eV, the DCS dependences on the scattering angle are presented

in figure 2. A significant deviation of the finite nucleus potential from the pure Coulomb potential exists at very small distances which have the order of the nuclear size. In the scattering amplitude it affects about seven first angular momenta only. Thus the DCS for the finite nucleus potential is close to the Rutherford cross section and does not show any maximum at 180° . For the same reason, the strong interaction between the antiproton and the nucleus is not important for the Coulomb glory effect either, and we include only electromagnetic interaction in the scattering potential (8). As one can see from figure 2, the most important part of the total scattering potential is the Uehling potential which is responsible for the DCS maximum at 180° . Although the Wichmann–Kroll potential has a longer range than the Uehling potential, its absolute value is much smaller. The effect of the Wichmann–Kroll potential results in a slight decrease of the DCS in the vicinity of $\theta = 180^\circ$. We emphasize that in the case under consideration the Coulomb glory effect results exclusively from the screening property of the VP potential. This property means a decrease of the effective charge $Z_{\text{eff}}(r) \equiv -rV(r)$ with r increasing. In figure 3 we plot the effective charge Z_{eff} versus the distance r for the total scattering potential as well as for the finite nucleus potential only. Except for the small region inside the nucleus, the effective charge for the total potential smoothly decreases from the maximum value of 92.32 to the uranium charge number 92. For the finite nucleus potential, the effective charge quickly approaches the value 92 just outside the nucleus.

In summary, we have investigated the elastic scattering of low-energy antiprotons by bare uranium nuclei at large angles and found that the differential cross section has a maximum in the backward direction (the Coulomb glory). The largest effect is predicted for the antiproton energy of about 300 eV. The existence of the Coulomb glory requires a screened Coulomb potential and, therefore, the phenomenon revealed is due to the screening properties of the vacuum polarization potential. Possible experimental observation of the Coulomb glory in collisions of antiprotons with bare uranium nuclei at the new GSI facility can become a clear manifestation of the vacuum polarization effects in non-relativistic collisions of heavy particles.

Acknowledgments

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